Unsteady and transient flow of compressible fluids in pipelines—a review of theoretical and some experimental studies

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The mathematical modelling of highly compressible unsteady flows has been of interest for some years. In order to obtain tractable solutions of the governing equations, investigators have made various simplifying assumptions such as presuming isothermal or isentropic flow of ideal gases, etc. The present review, with dense phase gas transmission systems of particular interest, briefly develops the basic equations without such assumptions and includes the effects of wall friction and heat transfer. After re-expressing the equations in terms of the measurable quantities of pressure, temperature and velocity, previously published work is reviewed for their solution. Relevant experimental work is somewhat limited but contributions from 20 references are included.

Keywords: *unsteady compressible flow, gas pipelines, fluid transients*

Introduction

The scale and complexity of modern gas recovery and pipeline distribution systems is such that they seldom, if ever, operate under steady flow conditions. Their design and simulation should always be on the premise that the flow is unsteady.

For analysis purposes, such flow situations may be divided into two main categories: slow and rapid transients. Slow transients are those fluctuations in pressure and flow caused by changes in demand: for example, on a daily cycle. They are mainly concerned with the packing and unpacking of gas in the system.

Rapid transients are those caused by a linebreak (pipe rupture), compressor failure, rapid shut-down or start-up of a system. The detection of linebreaks can be important both from an economic and a safety point of view. Although a linebreak is unlikely to occur through technical reasons, the risk of accidental pipe rupture cannot be ignored. The potential hazard arising from such a situation can be estimated by examining pipeline flow and pressure upstream of a suddenly open pipe.

Basic equations for homogeneous compressible flow

Since the application being considered is highly compressible flow in a pipeline, a control volume of length dx and an area equal to that of the pipeline is defined. Applying the laws of conservation of mass, of linear momentum and of energy leads to the basic equations for homogeneous, geometrically one-dimensional flow. This initial assumption is examined in detail elsewhere[1], but briefly it may be stated that for high Reynolds number flows (as in gas transmission lines) the one-dimensional assumption has been shown to be very good for steady and slowly varying flows. There may, however, be some slight deviations for large, rapid disturbances.

In terms of partial differential equations, the conservation laws may be expressed as follows:

for mass

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)=0 \tag{1}$$

for linear momentum

$$\rho\frac{\partial u}{\partial t}+\frac{\partial p}{\partial x}+\rho u\frac{\partial u}{\partial x}=-\frac{W}{A}-\rho g\sin\theta \tag{2}$$

for energy

$$\rho\frac{\partial h}{\partial t}+\rho u\frac{\partial h}{\partial x}-\frac{\partial p}{\partial t}-u\frac{\partial p}{\partial x}=\frac{\Omega+Wu}{A} \tag{3}$$

Eqs (1) to (3) may be rewritten[2] with pressure, velocity and temperature as the dependent variables by using the equation of state for a real gas:

$$P=z\rho RT$$

and the thermodynamic identity given by Zemansky[3]:

$$\mathrm{d}h=C_p\,\mathrm{d}T+\left\{\frac{T}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P+1\right\}\frac{\mathrm{d}P}{\rho}$$

The following set of hyperbolic equations is produced:

$$\frac{\partial P}{\partial t}+u\frac{\partial P}{\partial x}+\rho a_s^2\frac{\partial u}{\partial x}=\frac{a_s^2}{C_PT}\left\{1+\frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P\right\}\frac{\Omega+Wu}{A} \tag{4}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{1}{\rho}\frac{\partial P}{\partial x}=-\frac{W}{A\rho}-g\sin\theta \tag{5}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+\frac{a_s^2}{C_p}\left\{1+\frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P\right\}\frac{\partial u}{\partial x}=\frac{a_s^2}{C_pP}\left\{1-\frac{P}{z}\left(\frac{\partial z}{\partial P}\right)_T\right\}\frac{\Omega+Wu}{A} \tag{6}$$

The complete derivation of the above equations is given in the Appendix.

Methods of solution

Several different methods of solution for the general equations have been developed, and the choice is partly dependent upon

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the individual requirements of the system under investigation.

The more popular methods of solutions used for transient pipeline analysis are summarized below.

The method of characteristics

The method of characteristics simplifies the hyperbolic equations describing the flow (Eqs (4), (5) and (6)) by converting them to the natural coordinates of the system, otherwise known as the characteristics. The resulting characteristic equations obtained are solved numerically on either a grid of characteristics or on a rectangular coordinate grid.

The equations describing the flow may be written in matrix form thus:

$$\mathbf{U}_t + \mathbf{A}\mathbf{U}_x + \mathbf{d} = 0 \tag{7}$$

where subscripts t and x denote partial derivatives with respect to time and distance, respectively, and where

$$\mathbf{U} = \begin{pmatrix} P \\ u \\ T \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} u & \sigma a_s^2 & 0 \\ 1/\rho & u & 0 \\ 0 & \dfrac{a_s^2}{C_p}\left\{1 + \dfrac{T}{z}\left(\dfrac{\partial z}{\partial T}\right)_P\right\} & u \end{pmatrix}$$

$$\mathbf{d} = \begin{pmatrix} -\dfrac{a_s^2}{C_p T}\left\{1 + \dfrac{T}{z}\left(\dfrac{\partial z}{\partial T}\right)_P\right\}\dfrac{\Omega + Wu}{A} \\ \dfrac{W}{A\rho} + g\sin\theta \\ -\dfrac{a_s^2}{C_p P}\left\{1 - \dfrac{P}{z}\left(\dfrac{\partial z}{\partial P}\right)_T\right\}\dfrac{\Omega + Wu}{A} \end{pmatrix}$$

The eigenvalues λ of $\mathbf{A}$ give the characteristic directions, which are

$$\lambda_1 = u$$

$$\lambda_2 = u + a_s$$

$$\lambda_3 = u - a_s$$

The characteristic equations are as follows:

along $\dfrac{dx}{dt} = u$:

$$-\frac{1}{\rho C_p}\left\{1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P\right\}\frac{dP}{dt} + \frac{dT}{dt} - \frac{1}{C_p P}\left(\frac{\Omega + Wu}{A}\right) = 0 \tag{8}$$

along $\dfrac{dx}{dt} = u + a_s$:

$$\frac{1}{\rho a_s}\frac{dP}{dt} + \frac{du}{dt} - \frac{a_s}{\rho C_p T}\left\{1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P\right\}\left(\frac{\Omega + Wu}{A}\right) + \frac{W}{A\rho} + g\sin\theta = 0 \tag{9}$$

along $\dfrac{dx}{dt} = u - a_s$:

$$-\frac{1}{\rho a_s}\frac{dP}{dt} + \frac{du}{dt} + \frac{a_s}{\rho C_p T}\left(1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P\right)\left(\frac{\Omega + Wu}{A}\right) + \frac{W}{A\rho} + g\sin\theta = 0 \tag{10}$$

The method of solving these characteristic equations on a grid of characteristics is known as the *natural method of characteristics*.

For two dependent variables (as in the case of isothermal flow) the characteristic equations may be given by

$$\frac{du}{dt} \pm \frac{1}{\rho a}\frac{dP}{dt} + \frac{W}{A} + g\sin\theta = 0$$

$$\text{along } \frac{dx}{dt} = u \pm a \tag{11}$$

A first-order finite difference approximation to the C^+ characteristic gives (referring to the notation of Fig 1)

$$(u_B - u_{A1}) + \frac{1}{\rho a_{A1}}(P_B - P_{A1}) + \left(\frac{W}{A} + g\sin\theta\right)_{A1}(t_B - t_{A1}) = 0 \tag{12}$$

and

$$(x_B - x_{A1}) = (u_{A1} + a_{A1})(t_B - t_{A1}) \tag{13}$$

Notation

A	Cross-sectional area of pipe, m^2
a_s	Isentropic wavespeed, m/s
C_p	Specific heat at constant pressure, J/kg K
C_v	Specific heat at constant volume, J/kg K
d	Diameter of pipe, m
e	Specific internal energy, J/kg
f	Darcy friction factor
g	Gravitational acceleration, m/s^2
h	Specific enthalpy, J/kg
i, j	Rectangular coordinates used in explicit finite difference methods
P	Pressure, Pa
Pr	Prandtl number
Q	Heat transfer rate per unit volume, $J/(m^3 s)$
R	Specific gas constant, J/kg
Re	Reynolds number
s	Specific entropy, J/(kg K)
St	Stanton number
T	Temperature of the gas, K
t	Time, s
u	Velocity of the gas, m/s
W	Frictional force per unit length of pipe, N/m
x	Distance along the pipe, m
x'	Thermodynamic quality or dryness fraction
z	Gas compressibility factor
θ	Angle of inclination of pipe to the horizontal, radian
ρ	Mean density of the gas, kg/m^3
Ω	Heat flow into the pipe per unit length of pipe and per unit time, J/(m s)

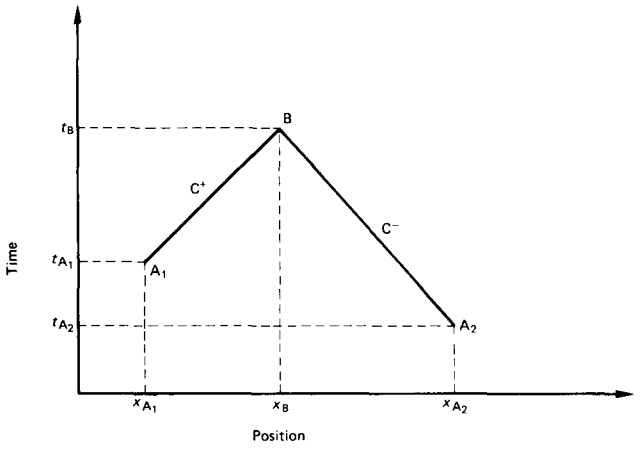


Figure 1 Linear characteristics on an x–t plane

Similarly for the C^- characteristics:

$$(u_B - u_{A2}) - \frac{1}{\rho a_{A2}}(P_B - P_{A2}) + \left(\frac{W}{A} + g\sin\theta\right)_{A2}(t_B - t_{A2}) = 0 \quad (14)$$

and

$$(x_B - x_{A2}) = (u_{A1} - a_{A2})(t_B - t_{A2}) \quad (15)$$

Eqs (12) to (15) can be solved simultaneously[4] for the four unknowns (P_B, u_B, x_B, t_B).

Instead of linearizing the characteristic grid, a second-order approximation may be used, as expressed by the trapezoidal rule formula. This results in a set of nonlinear equations which may be solved by iteration. Higher-order methods have been constructed[5] but because the number of points to be considered grows exponentially with distance from the line of known values I (see Fig 2), the range of applicability is limited.

The main advantage of the natural method of characteristics is that discontinuities can be handled and that large time steps are possible since they are not restricted by a stability criterion. However, this method does have two main disadvantages when dealing with rapid gas transients. The first is that if more than two dependent variables are required to describe the system then the complexity of the computation increases. The second major drawback is that if the solutions of the dependent variables are required at fixed time intervals, then two-dimensional interpolation in the characteristic net is required, and this can be very complicated. To overcome this second disadvantage, the *mesh method of characteristics* was developed which solves the characteristic equations on values for the dependent variables at specified time–distance coordinates.

A first-order method[6] assumes that the sections of the characteristics being considered are straight lines. This assumption is acceptable provided that the time steps, Δt, are sufficiently small. The interpolations are normally linear in the space domain. For larger time steps, discretization errors may be introduced if the characteristic lines are assumed to be straight. To overcome this, a second-order approximation is necessary which uses arcs of parabolas to model the characteristics.

An extension of this method for calculating three dependent variables, as required for transient non-isothermal gas flow, has also been successfully evolved[7–9].

Lister[10] describes a second-order method which obtains a higher degree of accuracy for smooth functions by using quadratic instead of linear interpolation. However, since there are three simultaneous equations to solve at each iterative step, the computing time would be increased. This method also causes comparatively large overshoot resulting from discontinuities.

Another way of increasing the accuracy of the solution is to use extrapolation procedures[9,11] which enable the elimination of higher-order errors (again at the expense of increased computing time).

Whereas the natural method of characteristics is unconditionally stable, the mesh method of characteristics is only conditionally stable. The stability criterion, due to Courant–Friedrichs–Levy, is that the domain of dependence of the exact solution is contained within the domain of dependence of the numerical solution[12]. In terms of mesh dimensions:

$$\frac{\Delta t}{\Delta x} \leqslant \frac{1}{|u| + a_s}$$

If the Courant number α is defined by

$$\alpha = (|u| + a_s)\frac{\Delta t}{\Delta x}$$

then this stability criterion can be given by

$$\alpha \leqslant 1$$

There are, however, certain circumstances in which adherence to the stability criterion could cause numerical dispersion of the waves. For example, problems arise when the absolute gradients of the C^+ and C^- characteristics differ significantly from each other (as would occur with high Mach numbers) or when the wavespeed varies significantly along the length of the pipe. In order to overcome such difficulties, Vardy[13] has proposed a method in which a variable mesh size is used. He concludes that in certain circumstances, such as high Mach number flows, increased accuracy and/or reduced computing costs can be obtained if $\Delta t/\Delta x$ grid ratios in excess of those permitted by the Courant–Friedrichs–Levy criterion are used, provided that the flow parameters at the base of the characteristic lines are still found by interpolation, rather than extrapolation.

Another method of relaxing the stability criterion is by using an inertial multiplier α as conceived by Yow[14]. By assuming that the inertial effect in a natural gas system is insignificant, Yow multiplied the term $(\partial u/\partial t)$ by α^2. This increased the time step by a factor α. The choice of α (not to be confused with the Courant number α) is dependent on the severity of the transient being examined and the accuracy required. Wylie and Streeter[15] illustrate that with a 5% error margin, the time step may be increased by a factor of 6 for a rapid transient or by a factor of 40 for a slow transient.

With the mesh method of characteristics, discontinuities can be handled and boundary conditions are properly posed. It is a relatively accurate method of solution which can be readily

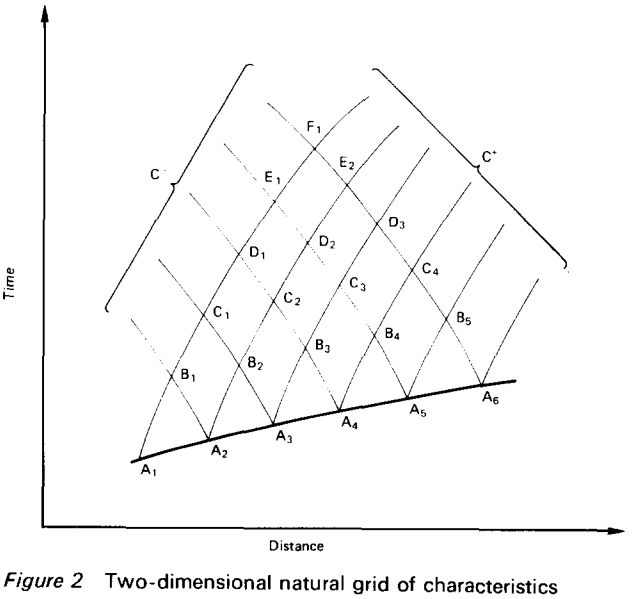


Figure 2 Two-dimensional natural grid of characteristics

adapted to solve for the three dependent variables required for the analysis of nonisothermal, transient gas flow. It is simple to program on a computer, although the main disadvantage is that it is comparatively slow because the time steps are restricted by a stability criterion.

Explicit finite difference methods

There are many different explicit finite difference methods, ranging from the single-step, first-order schemes, such as the method of Lax (p 85 of Ref 16), to the fourth-order, four-step method of Abarbanel, Gottlieb and Turkel[17]. Second-order accuracy is normally regarded as sufficient for the analysis of gas transients. Niessner[18] gives details of higher-order methods.

Explicit finite difference methods integrate the basic partial differential equations by considering the changes in the dependent variables (P, u and T) along the directions of the independent variables (x and t). This produces the solution values at evenly spaced points in the physical plane. A finite difference grid is shown in Fig 3.

To solve the basic equations using an explicit finite difference method they should first be written in the 'conservative' form[19]

$$\frac{\partial}{\partial t}(A)+\frac{\partial}{\partial x}(B)=C \tag{16}$$

where A, B and C are functions of the dependent variables.

For the case of transient gas flow in pipes, the three conservation equations (Eqs (1), (2) and (3)) may be written thus:

MASS

$$\frac{\partial}{\partial t}(\rho)+\frac{\partial}{\partial x}(\rho u)=0 \tag{17}$$

MOMENTUM

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}(\rho u^2+P)=-\frac{W}{A}-\rho g\sin\theta \tag{18}$$

ENERGY

$$\frac{\partial}{\partial t}\left[\left(e+\frac{u^2}{2}\right)\rho\right]+\frac{\partial}{\partial x}\left[\left(h+\frac{u^2}{2}\right)\rho u\right]=\Omega-\rho u g\sin\theta \tag{19}$$

The simplest explicit finite difference method is the forward Euler method. Applying this method to Eq (16) (assuming that C is equal to zero) produces the following approximation:

$$A_{(i,j+1)}=A_{(i,j)}-\frac{\Delta t}{2\Delta x}(B_{(i+1,j)}-B_{(i-1,j)}) \tag{20}$$

This method is unconditionally unstable, and to overcome this a damping term must be added to produce

$$A_{(i,j+1)}=A_{(i,j)}-\frac{\Delta t}{2\Delta x}(B_{(i+1,j)}-B_{(i-1,j)})+\frac{\omega}{4}(A_{(i+1,j)}-2A_{(i,j)}+A_{(i-1,j)}) \tag{21}$$

where $0<\omega\leqslant 2$ and is the natural frequency of the oscillations. This is known as the 'Method of Lax' and is a single-step, first-order method.

In general, a first-order approximation is not sufficiently accurate for modelling gas transients in pipelines, and so attention is focused on the second-order methods. A single-step second-order finite difference method is the 'Method of Lax–Wendroff'[19] which can be written as

$$A_{(i,j+1)}=A_{(i,j)}-\frac{\Delta t}{2\Delta x}(B_{(i+1,j)}-B_{(i-1,j)})+\frac{1}{4}\left(\frac{\Delta t}{\Delta x}\right)^2\left\{\left(\frac{\partial B}{\partial A_{(i+1,j)}}+\frac{\partial B}{\partial A_{(i,j)}}\right)(B_{(i+1,j)}-B_{(i,j)})-\left(\frac{\partial B}{\partial A_{(i,j)}}+\frac{\partial B}{\partial A_{(i-1,j)}}\right)(B_{(i,j)}-B_{(i-1,j)})\right\} \tag{22}$$

This method has the disadvantage that additional computing time is required to evaluate $\partial B/\partial A$ as well as B at each step. To avoid the necessity of this calculation there have been numerous two-step methods developed. Probably the best-known of these is the 'Lax–Wendroff two-step'. This method was used to simulate dynamic gas flows in networks[20] and to simulate pressure wave propagation in two-phase bubbly air–water mixtures[21]. Taking Eq (16), the Lax–Wendroff two-step approximation may be described as follows:

First step

$$A_{(i+1/2,j+1/2)}=\frac{1}{2}[A_{(i+1,j)}+A_{(i,j)}]-\frac{1}{2}\frac{\Delta t}{2\Delta x}[B_{(i+1,j)}-B_{(i,j)}]+\frac{\Delta t}{2}[C_{(i+1,j)}+C_{(i,j)}]+O(\Delta x^2,\Delta t) \tag{23}$$

Second step

$$A_{(i,j+1)}=A_{(i,j)}-\frac{\Delta t}{\Delta x}[B_{(i+1/2,j+1/2)}-B_{(i-1/2,j+1/2)}]+\Delta t[C_{(i+1/2,j+1/2)}+C_{(i-1/2,j+1/2)}]+O(\Delta x^2,\Delta t^2) \tag{24}$$

where $O(\Delta x^2,\Delta t^2)$ is the 'truncation' or 'rounding' error. On close examination of these equations, it can be seen that in the first step the values at all the points at time $t=j+\frac{1}{2}$ can be found. These values are then used in the second step to derive the values at time $t=j+1$.

The MacCormack method[22] is also a second-order two-step method. This is described by

First step

$$\bar{A}_{(i,j+1)}=A_{(i,j)}-\frac{\Delta t}{\Delta x}[B_{(i+1,j)}-B_{(i,j)}]$$

Second step

$$A_{(i,j+1)})=\frac{1}{2}[A_{(i,j)}+\bar{A}_{(i,j+1)}]-\frac{\Delta t}{2\Delta x}[\bar{B}_{(i,j+1)}-\bar{B}_{(i-1,j+1)}]$$

Another second-order method is the 'leap-frog' method[24]. This method involves three time levels within one time step, and the approximation for Eq (16) (assuming that C is equal to zero)

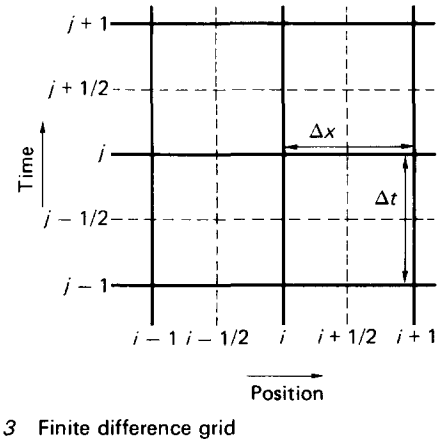


Figure 3 Finite difference grid

may be written

$$A_{(i,j+1)}=A_{(i,j-1)}-\frac{\Delta t}{\Delta x}[B_{(i+1,j)}-B_{(i-1,j)}] \quad (25)$$

This method shows no amplitude error and requires only one evaluation of the value for B at each node point. However, when C in Eq (16) is not equal to zero, this method becomes unconditionally unstable, and to regain stability the calculations become more complicated.

A major drawback of the explicit finite difference methods is that, at best, they are only conditionally stable. For most cases the stability criterion is the same as that defined for the mesh method of characteristics, ie

$$\frac{\Delta t}{\Delta x}\leqslant\frac{1}{|u|+a_s}$$

Since the stability criterion restricts the size of time step which may be used, these methods require a large amount of computer time and are hence not suitable for the analysis of large systems or for the evaluation of unsteady flows over long periods of time. They are, however, easy to program and need comparatively little computer memory space since they solve the equations sequentially rather than simultaneously.

For systems in which a shock forms, an explicit finite difference method would be suitable since no care needs to be taken over the location of the shocks. To overcome the considerable overshoot and oscillatory systems set up by the shock when using a method of higher than first order, a smoothing parameter is used. However, extreme care must be taken when using such numerical damping since it can tend to smoothen the transient peaks.

A disadvantage of these methods is their inability to solve for the boundary conditions naturally. In some cases the boundary conditions are solved using the method of characteristics but the calculations may be complicated for networks with many branches.

Implicit finite difference methods

Implicit finite difference methods have the advantage over the explicit methods of being unconditionally stable. This implies that the maximum practical time step is limited by the rate of change of the variables imposed at the boundary conditions rather than by a limitation required by a stability criterion. Some of the implicit finite difference methods that have been used in the solution of fluid transient problems are detailed below. The notation used for each method is that illustrated in Fig 4.

Fully implicit method

This method is a backward difference method (whereas the explicit finite difference schemes are forward difference methods). For the general equation in conservative form (Eq (16)) the fully implicit finite difference approximation for the point (c, 1) may be written

$$\frac{A_{c1}-A_{c0}}{\Delta t}+\frac{B_{d1}-B_{b1}}{2\Delta x}=C_{c1} \quad (26)$$

The node points used in this approximation are shown in Fig 5(a).

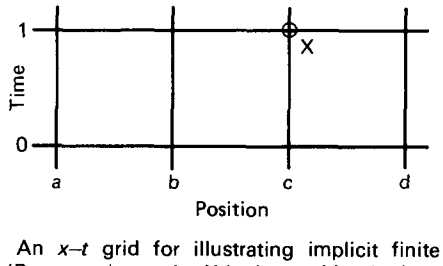


Figure 4 An x–t grid for illustrating implicit finite difference methods. (Property ϕ at point X is denoted by ϕ_{c1}.)

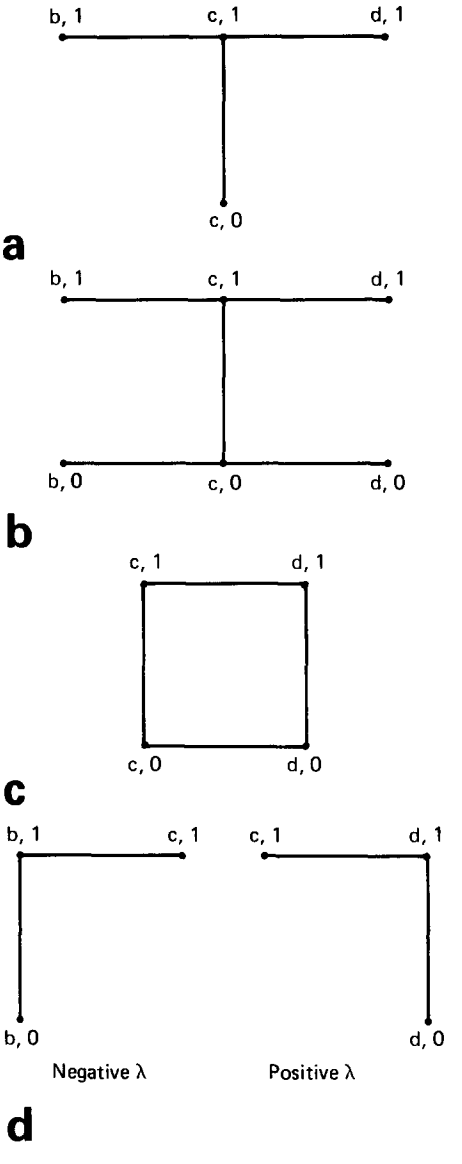


Figure 5 Definition sketches for fully implicit methods: (a) the fully implicit method; (b) the Crank–Nicolson method; (c) the centred difference method; (d) the characteristic finite difference method

The Crank–Nicolson method

Forsythe and Wasow[16] reported that the implicit difference methods 'seem to have been used for the first time by Crank and Nicolson (1947)'. What is now known as the Crank–Nicolson method is a central difference solution of high-order accuracy. This solution is, however, prone to oscillate about the true solution for sudden changes in forcing function. The Crank–Nicolson approximation for Eq (16) at the point (c, 1) is

$$\frac{A_{c1}-A_{c0}}{\Delta t}+\frac{(B_{d0}-B_{b0})+(B_{d1}-B_{b1})}{4\Delta x}=C_{c1} \quad (27)$$

Fig 5(b) gives the nodal plan for this method. Guy[23] and Heath and Blunt[24] used the Crank–Nicolson method to solve the conservation of mass and the conservation of momentum equations for slow transients in isothermal gas flow. Both research teams neglected the elevation term $\rho g \sin\theta$) and the

differential of kinetic energy with distance ($\partial/\partial x(\rho u^2)$) in the momentum equation (Eq (18)).

The justification for these omissions is that the relative orders of magnitude of the terms $\partial/\partial x(\rho u^2):\partial/\partial t(\rho u):\rho P/\partial x$ are approximately 0.01:0.1:1 so it is reasonable to neglect the nonlinear term $\partial/\partial x(\rho u^2)$. The elevation term is often considered to be insignificant but is retained here for completeness.

This method of solution was found to be much simpler than those proposed by Wilkinson *et al*[25]. It was easier to program, computed much faster and could be readily extended to pipeline networks of any size.

The centred difference method

Wylie *et al*[26] used the centred difference method to solve for isothermal gas transients in a network. In this method the partial derivatives are calculated for sections of the pipeline rather than node points. For section c–d in Fig 4 the centred difference approximation for Eq (16) is

$$\frac{(A_{d1}-A_{d0})+(A_{c1}-A_{c0})}{\Delta t}+\frac{(B_{d1}-B_{c1})+(B_{d0}-B_{c0})}{\Delta x}=\frac{C_{c1}+C_{c0}+C_{d1}+C_{d0}}{2} \tag{28}$$

The node points used in this approximation are shown in Fig 5(c).

Although this method does require a large amount of computer storage to handle the coefficient matrix and lengthy execution times these major disadvantages can be overcome by using a sparse matrix method.

A development of this method incorporating upstream weighting was used by Taylor[27]. This weighted finite difference approximation for Eq (16) at points P as shown in Fig 6 is given by

$$\frac{\theta(A_{d1}-A_{d0})+(1-\theta)(A_{c1}-A_{c0})}{\Delta t}+\frac{\phi(B_{d1}-B_{c1})+(1-\phi)(B_{d0}-B_{c0})}{\Delta x}=(1-\phi)\{\theta C_{d0}+(1-\theta)C_{c0}\}+\phi\{\theta C_{d1}+(1-\theta)C_{c1}\} \tag{29}$$

where θ and ϕ are the weighting factors

$0\leqslant\theta\leqslant 1,\quad 0\leqslant\phi\leqslant 1$

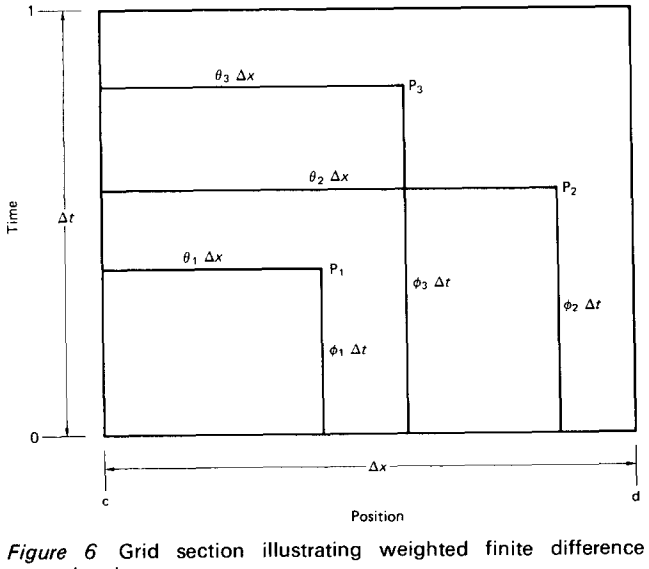


Figure 6 Grid section illustrating weighted finite difference approximations

Characteristic finite difference method

The characteristic finite difference method can be used[28,29] to simulate transient homogeneous two-phase flow. It is so called because instead of approximating the conservative form of the basic equations (Eq (16)) it uses the characteristic form of the equations. The characteristic form may be written

$$\mathbf{T}\frac{\partial \mathbf{u}}{\partial t}+\mathbf{\Lambda T}\frac{\partial \mathbf{u}}{\partial x}=\mathbf{D} \tag{30}$$

where $\mathbf{T}$ is the transformation matrix, $\mathbf{u}$ is the column vector of the dependent variables, and $\mathbf{\Lambda}$ is the diagonal matrix of the characteristic directions.

The difference approximations at point (c, 1) of Fig 4 for Eq (30) may be written

$$\mathbf{T}_{c0}\left(\frac{\mathbf{u}_{c1}-\mathbf{u}_{c0}}{\Delta t}\right)+\mathbf{\Lambda}_{c0}\mathbf{T}_{c0}\left(\frac{\mathbf{u}_{c1}-\mathbf{u}_{b1}}{\Delta x}\right)=\mathbf{D}_{c0} \tag{31}$$

$$\mathbf{T}_{c0}\left(\frac{\mathbf{u}_{c1}-\mathbf{u}_{co}}{\Delta t}\right)+\mathbf{\Lambda}_{c0}\mathbf{T}_{c0}\left(\frac{\mathbf{u}_{d1}-\mathbf{u}_{c1}}{\Delta x}\right)=\mathbf{D}_{c0} \tag{32}$$

Eq (31) is used for the positive characteristic directions, and Eq (32) is used for the negative characteristic directions. The relevant node points are shown in Fig 5(d). This method is as similar as possible to the method of characteristics but is not restricted by a stability criterion.

General review of implicit finite difference methods

The four methods that have been described are the implicit finite difference methods most commonly used for gas transient analysis, although there are others such as the explicit–implicit methods[30] used to solve for pressure transients in bubbly two-phase mixtures or the three time level implicit scheme discussed by Osiadacz[31].

The major advantage of using an implicit finite difference method is that they are unconditionally stable and hence impose no restrictions on the maximum allowable time step. These methods do, however, require the solution of a set of nonlinear simultaneous equations (usually by Newton–Raphson linearization) at each time step. For a complex gas network the matrix becomes quite large. Other disadvantages of these methods of solution are that they can yield unsatisfactory results for sharp transients and some implicit methods have been known to produce erratic results during the imposition of some types of boundary condition.

Finite element analysis

Finite element methods have not been widely used for gas transients since the procedure is lengthy and tedious, and computing time and storage requirements are high. However, they do offer some advantages over the finite difference methods in that the element size, shape and distribution are relatively flexible so that nonuniform internal distribution of nodal points is possible. They can also handle some boundary conditions better than finite difference methods.

The various steps involved in the finite element method of solution are as follows.

(i) Subdivision of the pipeline into subregions or finite elements—the size, shape and distribution must be decided.

(ii) Selection of the shape functions—the dependent variables may be approximated by different shape functions in each element. The shape functions are usually polynomials, the simplest of which is the linear or chapeau representation. The higher order polynomials yield more accurate solutions unless the solution contains discontinuities, in which case this does not always hold true.

(iii) Derivation of element behaviour—a relationship is obtained for a typical element, and from this the behaviour of all the individual elements may be computed.

(iv) Application of the boundary conditions—the boundary conditions are applied by modifying the overall algebraic equations.
(v) Solution of the overall equations—these equations, especially when nonlinear, are usually solved iteratively.

Rachford and Dupont[32] used a Galerkin finite element method with two-dimensional elements in space–time based on Hermite Cubic polynomials to simulate isothermal transient gas flow. The Galerkin method is a two-step method which reduces the partial differential equations to ordinary differential equations, but it is generally unpopular for this type of analysis because of its lengthy execution.

In the moving finite element method of solution[33,34] ordinary differential equations are obtained from the basic partial differential equations and then solved using a simple explicit finite difference approach, using one-dimensional elements in space with linear differences in the time domain. The major drawbacks with this method of solution are that care is needed in the treatment of the boundary conditions and that it is very complicated to program.

The flux difference splitting schemes

Recent developments in the solution of the basic equations for transient gas flow have included

(i) λ formulation, proposed by Moretti[35] in 1979;
(ii) flux-vector splitting, proposed by Steger and Warming[36] in 1981;
(iii) flux-difference splitting, developed by various authors during the last five years from Godunov's[37] work of 1959.

The λ formulation scheme has the major drawback that shock waves have to be treated explicitly. Mulpuru[38] showed that the flux-vector splitting techniques of first order are very diffusive whereas higher-order methods generate post shock oscillations. The problem can, however, be overcome by using a nonlinear weighting procedure[39] which produces a hybrid scheme which can be extended to higher spatial dimensions through time splitting.

The flux-difference splitting schemes can correctly capture the shock waves and provide criteria to discriminate the correct information carried by propagating waves. The difference in flux between two adjacent node points is split into terms that will affect the flow evolution at points either side of the section under investigation. It is assumed that uniform flow occurs at each node points and over the cell extending for one half grid interval each side of the node point. A discontinuity generally separates two neighbouring cells in the middle of the interval, and the evolution in time of this discontinuity provides the criteria for splitting the flux difference over an interval into terms associated with waves that propagate up or down the pipe. Roe[40] describes this method of solution for the basic equations with the source terms omitted, and Pandolfi[41] extends the analysis to hyperbolic equations.

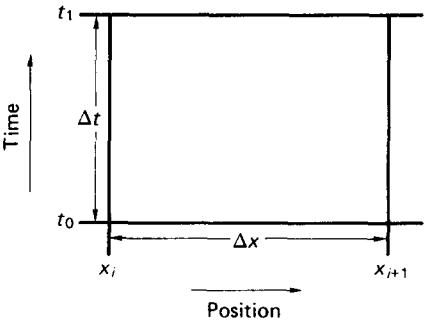
With reference to the elemental section shown in Fig 7, let

$$B_{i+1} - B_i = \Delta_i B$$

The term $\Delta_i B$ is known as the flux difference, and the corresponding term $\Delta_i B\,\Delta t/\Delta x$ can be interpreted as the contribution of the interval $(x_{i+1} - x_i)$ to the variation in time, from t_0 to t_1, of the vector A. In general the waves will travel in both directions in the pipeline and so it is necessary to split the term $\Delta_i B$ into parts that will affect the points upstream or downstream of the interval under consideration.

At time t_0 let there be uniform flow B_i in the interval $(x_{i+1/2} - x_i)$ and uniform flow B_{i+1} in the interval $(x_{i+1} - x_{i+1/2})$. A discontinuity ($\Delta_i A$ and/or $\Delta_i B$) separates the two half intervals at the centre $(x_{i+1/2})$. The evolution in time of this discontinuity is the solution of a Riemann problem.

Since for this problem there will be three waves corresponding to the three characteristic directions $(u+a, u, u-a)$ the



Section 7 Elemental section for flux difference splitting

difference of flux through the initial discontinuity $\Delta_i B$ is split into three terms:

$$\Delta_i B = (\Delta_i B)_1 + (\Delta_i B)_2 + (\Delta_i B)_3$$

P. L. Roe[40] reported that the exact solution of this Riemann problem is not essential to obtain good numerical results, especially considering the large truncation errors that would be incurred in the iterative process required to obtain the exact solution. Instead, the Riemann problem is solved approximately to save on computing time, and it is the different ways of approximating that identify the different flux-splitting methods.

Although very good results have been obtained from numerical experimentation, these methods do have the basic disadvantage that a considerable amount of computer time is required to split the flux difference. Furthermore, if a second-order method is used for the integration the computational time is again increased. Also it has been noted that some inaccuracies can develop in cases such as the interaction of shocks.

Further comments on the basic equations

The basic equations contain certain terms, such as the friction term and the heat transfer term, that require further clarification before numerical results can be obtained.

Friction term

The friction term, denoted by W in the basic equations, may be defined as 'the frictional force per unit length of pipe' opposing the flow. Assuming that the minor losses are small compared with the distributed losses, the frictional force W for a gas may be written

$$W = \frac{A}{d}\rho f \frac{u|u|}{2} \tag{33}$$

where f is the Darcy friction factor.

Various research teams have used different relationships to define the friction factor, and several questions arise, as discussed below.

(*i*) *Can steady flow friction factors be applied to unsteady flow?* Since, at present, there have been no friction factors defined for transient gas flows, it is common practice to use the steady flow definitions in lieu. Some time-dependent friction factors have been developed for laminar liquid flows[42–44] but these would not be suitable for turbulent gas flows. When the transient flow is of relatively low frequency and amplitude, there is very little error involved in using a steady flow friction factor. However, if large, rapid disturbances are occurring, a significant error may be incurred. For this reason, 'tuning' of the friction term may be employed when investigating rapid transients.

(*ii*) *Is it necessary to use a flow-dependent friction factor?* The friction factor is dependent on pipe roughness and Reynolds number, which vary from point to point in gas pipelines. The

main consideration is whether the variation in friction factor is large enough to justify the additional computation involved. Henry[45] states that the friction factor may be determined to within 1 % in a high pressure pipeline and that variations with flow should be considered. Other authors[7,8,23,46], however, all claim that at higher Reynolds numbers the friction factor can be assumed to be constant, and support their claim with experimental data. In practice the basic data for determining the friction factor, even for steady flows, will rarely be known within a few percent, and a constant friction factor will usually be adequate as a first approximation.

(iii) Does the friction factor need to account for the possibility of the liqid phase being present? If so, how? In dense gases a certain amount of condensation must be expected to occur with depressurization. For such cases, the liquid volume fraction will be less than 0.1. The friction factor is strongly dependent on the liquid volume fraction and different calculation techniques give substantially different results.

There are two main options for obtaining a value for the friction in two-phase flow:

(a) modify the Reynolds number and roughness terms of the Colebrook Equation[47];
(b) include in the expression for friction a multiplier which is determined empirically[29,30,48-50].

The preferred method is to include a two-phase friction multiplier since this method is relatively simple and has already been adapted by various authors for analysing transient flow situations. Modifying the Reynolds number and roughness terms is more appropriate for steady flow analysis.

(iv) How can the friction term be approximated when solving the basic equations? Frictional loss is responsible for most of the change in pressure along a gas transmission pipeline. A linearized friction term does not adequately represent this high frictional effect in a gas and so a second-order approximation such as the trapezoidal rule must be used.

Heat transfer term

The heat transfer term Ω may be defined as 'the heat flow into the pipe per unit length of pipe and per unit time'. Although it is considerably smaller in magnitude than the friction term, it is still a necessary inclusion especially when considering long distance pipelines.

Typically, either an isothermal or an adiabatic approach has been adopted. For the case of slow transients caused by fluctuations in demand, it is often assumed that the gas in the pipe has had sufficient time to reach thermal equilibrium with constant-temperature surroundings. Similarly, when rapid transients are under consideration, it is assumed that the pressure changes occur instantaneously, allowing no time for heat transfer to take place between the gas in the pipe and the surroundings. These are two extreme cases. In reality a certain amount of heat transfer will occur between the gas and its surroundings, although thermal equilibrium will not always be reached.

Various methods are available for estimating the heat transfer, most of which involve modifying a steady flow expression. One of the most popular of these methods is the use of the Stanton number which may be defined in terms of heat transfer rate:

$$St \equiv \frac{Qd}{4\rho C_p u(T_w - T_o)}$$

where Q is the heat transfer rate per unit volume, T_w is the wall temperature, T_o is the stagnation temperature.

Hence, for a circular cross-section pipe:

$$St = \frac{\Omega}{\pi\rho C_p u d(T_w - T_o)}$$

where Ω is the heat transfer rate per unit length of pipe as defined in the basic equations.

Therefore,

$$\Omega = \pi\rho C_p u St d(T_w - T_o)$$

The Stanton number may initially be found from boundary layer theory or taken as a function of the Reynolds and Prandtl numbers. For example, Bakhtar[51] used the relationship

$$St(Re^{0.2})(Pr^{0.6}) = \text{constant}$$

However, Issa and Spalding[7] concluded that, as with the friction factor, variations in Stanton number with flow rate were not sufficient to warrant the additional computation involved.

Equation of state

The compressibility factor z and its derivatives with respect to pressure and temperature appear in the basic equations. The compressibility factor may be read directly from a generalized compressibility chart but an alternative method is to use an equation of state. This has the advantage that it can be easily programmed into a computer and it can also solve for the derivatives of the compressibility factor. There is a wide selection of equations of state varying in accuracy and complexity. However, since the two terms $T/z(\partial z/\partial T)_P$ and $P/z(\partial z/\partial P)_T$ are usually relatively small, a complex equation of state would be uneconomical in terms of computer time.

Review of experimental work on dense gases and vapours

Relevant experimental work that has been carried out over the last forty years can be categorized into three types:

(i) laboratory work using shock tubes,
(ii) experimentation on full size pipelines,
(iii) experimentation using pipe networks.

Each form of investigation has its own advantages and disadvantages, and some research teams have used more than one type to support their theories.

Laboratory experiments

Rapid pressure transients can be modelled using shock tubes by rupturing a diaphragm or bursting disc. Edwards and O'Brien[52] used this method to simulate blowdown in a water-cooled power reactor. They heated a water-filled pipe to the required temperature and pressure (above saturation conditions) and then ruptured a glass bursting disc at the end of the tube. The transient pressures and temperatures were measured at seven tapping points along the length of the pipe, and transient void fraction readings were taken at two of the stations. The end thrust exerted by the shock tube was also measured. From their results they concluded that the pressure in the shock tube initially fell below and, although it recovered slightly, remained below the initial saturation value. Also the decompression wave, caused by the rupture of the bursting disc, travelled upstream at approximately the isentropic speed of sound in the compressed liquid phase.

In 1978, Groves *et al*[53] published some results they had obtained for pressure transients in gases. Until that time most of the experimental shock tube studies had investigated the transient effects with water vapour only and had been focused on the low pressure side of the diaphragm. Since Groves *et al* were attempting to simulate a gas pipeline rupture and to describe the decompression wave associated with such a rupture, they concentrated on the high pressure side of the diaphragm and used methane, argon and natural gas as working fluids. The results obtained illustrated the variation in wavespeed of the decomposition wave with pressure ratio.

Discrepancies between experimental and theoretical results were accounted for in that the small diameter effects (eg heat transfer, partial choking due to boundary layer build-up, and successive condensations) were not included in the theoretical analysis.

Issa and Spalding[7] obtained better agreement between their theoretical analysis and the experimental shock tube data of Williams[54] although they did not compare the theoretical and experimental variations of wavespeed with pressure ratio. The working fluid was assumed to be a perfect gas in the theoretical analysis, but frictional effects and heat transfer were included (which in practice weaken a shock wave).

In the experimental work discussed above, the fluid was initially at rest or moving with negligible velocity. Experiments performed by Premoli and Hancox[55] differed from other shock tube experiments in that the fluid was initially flowing when the rupture was initiated and also heat was added during the depressurization. Using subcooled pressurized water (steam–water) as their working fluid they produced data including depressurization rate, mass hold-up and discharge rate.

Full size pipeline experiments

Some authors, for example Cheeseman[56], argue that since rapid pressure transients are quickly reduced by friction, the main pressure transients of concern to the line operator are those arising from the packing and unpacking of gas in the pipeline. There have been several experimental studies on these slower transients, although, since they occur due to fluctuations in demand, it is more common to examine them using pipe networks.

Single pipelines are more frequently used to model rapid transients such as those caused by a linebreak or a rapid valve closure. In the first major series of experiments[25] five pipelines of various lengths, diameters and topography were examined. Flow and pressure variations were imposed at the outlet of each pipe and the flow and pressure were recorded at both ends of the pipe. Good agreement was found between theoretical and measured input flows and pressures for both rapid and slower transient conditions.

In the late 1960s, Stoner[46] determined the wavespeed of the compression wave caused by a rapid downstream valve closure in a 0.31 m diameter gas pipe. He then recorded the upstream and downstream pressure histories of the pipe when the valves at both ends of the pipe were simultaneously rapidly closed. Meanwhile, in France, Sens *et al*[57] were investigating the effects of rapidly opening a downstream valve to simulate an accidental pipeline break. It was found that at a distance of 6 km from the venting point the rapid opening of the discharge valve had the same effect as rupturing a bursting disc.

Rachford and Dupont[32] used a 0.59 m diameter, 53 km long, 2-leg gas pipeline and imposed sudden flow variations at the inlet end, slower variations at the outlet, and then compared the calculated and observed pressure histories.

Using a four-leg, 78 km long refinery gas transmission pipeline, Weimann[58] imposed transient supply and demand flows and compared the measured pressure variations with those predicted from his isothermal analysis. Although the changes in flow rate each took place within one minute, the resulting effect was the gradual packing and unpacking of the pipeline.

Recently (1983), Mekebel and Loraud[59] investigated unsteady flows and pressures in a 0.22 m diameter, 19.345 km long gas transmission pipeline operating at pressures below 20 bar. They concluded that heat transfer was a necessary inclusion in the theoretical analysis. This contradicts the common assumptions of isothermal or adiabatic flow.

In the Netherlands, experimental data from the Gasunie transport system was used[2] to validate a theoretical model. In the first of two experiments a linebreak was simulated by rapidly opening a valve which connected the test pipe to a parallel pipe at lower pressure. The point on the test pipe at which the measurements were taken was 10 km downstream of this valve. The second experiment involved rapidly opening a gate valve situated between two measuring points on a test pipe. The pipe was 90 km long with a diameter of 0.76 m. Gas was supplied at both ends of the pipe and delivered to a number of take-off points along the pipe. The results showed that a fast pressure transient occured at both measuring points due to the valve opening.

Pipe network experiments

Since variations in supply and demand in gas transmission networks produce slow pressure transients, many line operators are more interested in the analysis of these slower transients.

In one 7-hour test[24] on a section of a high pressure grid, flow rates and pressures were monitored at each take-off point and tee at 5-minute intervals following the isolation of the supply. Although the test was limited by having only one sudden flow change, the results obtained agreed well with those predicted from an isothermal analysis. It was realized, however, that just one inaccurate reading could affect the pressures predicted throughout the network, so extreme care had to be taken when recording the flow measurements.

Rachford and Dupont[32] compared predictions from their isothermal analysis with experimental data recorded over ten hours for slow transients in a complicated looped network. They obtained quite accurate pressure history predictions for various points around the network.

Weimann[58] used a branched network as well as the single 4-leg pipeline described above to validate his isothermal model predicting the packing and unpacking of the gas. He recorded supply and demand flows at 1-hour intervals and took pressure readings at 15-minute intervals for a 24-hour period.

Some experiments have been carried out using steam as the working fluid. These were conducted to support simulations for boiler steam lines and reactor blow-down.

Ying and Shah[60] investigated steam hammer in the main piping system of an oil-fired power plant. They imposed transient conditions in the network by rapidly closing the turbine stop valves and then obtained oscilloscope traces of the pressure surges created.

Banerjee and Hancox[28] conducted a series of blow-down experiments on a figure-of-eight loop containing pumps, heaters and heat exchangers. The blow-down was started by rapidly opening a quick-acting valve. Pressure, temperatures, coolant densities and flow rates were recorded at various points around the circuit, and the results obtained were compared with those predicted from the computer code of Arrison *et al*[61].

Concluding remarks

In reviewing the work of many authors it is evident that the basic equation set selected to describe a given unsteady flow process, and the numerical techniques adopted to yield solutions of practical use, usually involve compromises, eg accuracy may be traded off against computer memory space and speed of computation, etc. General recommendations may be summarized as follows.

(*i*) *Slow transient* (*time scale in hours*) *or rapid transients* (*time scale in seconds*)? If the analysis is solely concerned with slow transients, such as those caused by fluctuations in demand in a network, then considerable savings in computational time and hence cost will be made by utilizing an implicit finite difference scheme which does not require a small time step for stability. However, if rapid transients are being considered, such as those caused by a linebreak or compressor failure, the implicit finite difference methods produce unsatisfactory results. In this case, a small time step is required, and the method of characteristics is recommended.

(*ii*) *Degree of accuracy?* Higher degrees of accuracy can usually be achieved at the expense of increased computational labour. In general, the implicit finite difference schemes are more economical than the explicit finite difference schemes or the method of characteristics, although the latter can achieve more accurate results. When using the mesh method of characteristics, errors can be introduced if the characteristics are approximated to straight lines. These discretization errors can, however, be reduced by employing arcs of parabolas in place of the straight lines to give a second-order approximation.

(*iii*) Methods of solution that will not accommodate a varying wavespeed should not be used for nonisothermal flows. If shock waves develop in the system, a method must be chosen that will accurately represent the shock waves without smearing the details or overshooting. The Lax–Wendroff two-step explicit finite difference method is the most suitable for dealing with systems in which a shock wave forms. The natural method of characteristics is also accurate but requires special procedures for the shock calculations. The mesh method of characteristics or an extension of this method such as the flux difference splitting scheme[40] both recognizes shocks and cause only small overshoot. However, the finite difference methods tend to produce overshoot in the presence of shocks if methods of higher than first order are used, and the discontinuities tend to get rounded off due to numerical diffusion.

(*iv*) *Size of system—effect of boundary conditions.* Implicit finite difference methods are more suited to the analysis of large systems, although programs based on implicit methods do not allow easy extension. The mesh method of characteristics and the explicit finite difference approaches are comparatively slow and are more suited to single pipelines than to networks. However, the mesh method of characteristics does have the advantage that the boundary conditions are properly posed, whereas for most of the other methods of solution care is needed—some implicit methods have been known to produce erratic results during the imposition of some types of boundary condition.

(*v*) For virtually all situations of unsteady flow, and especially those involving rapid transients, there is ample scope to add to the pool of experimental data. This can be extremely costly to produce, but is nevertheless required to help validate computer codes. Measurements that lead to a better understanding of heat transfer and frictional effects for rapid transients are particularly scarce.

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References

1 Goldwater, M. H. and Fincham, A. E. Modelling of gas supply systems. In *Modelling of Dynamic Systems, Vol 2*, ed H. Nicholson, published by Peter Peregrinus Ltd on behalf of the Institute of Electrical Engineers, 1980

2 van Deen, J. K. and Reintsema, S. R. Modelling of high pressure gas transmission lines. *Appl. Math. Modelling*, August 1983, **7**

3 Zemansky, M. W. *Heat and Thermodynamics* 5th Edn, McGraw-Hill, 1968

4 Courant, R. and Friedrichs, K. O. *Supersonic Flow and Shock Waves*, Interscience Publishers, New York, 1948

5 Ansorge, R. Die Adams-Verfahren als Charakteristik-enverfahren hoherer Ordnung zur Losung von hyperbolischen Systemen halblinearer Differentialgleichungen. *Num. 7ath.* 1963, **5**, 443–460

6 Courant, R., Isaacson, E. and Rees, M. On the solution of nonlinear hyperbolic differential equations by finite differences. *Comm. Pure and Appl. Math.* 1952, **5**, 243–255

7 Issa, R. I. and Spalding, D. B. Unsteady one-dimensional compressible frictional flow with heat transfer. *J. Mech. Eng. Sci.* 1972, **14**(6), 365–369

8 Cronje, J. S., Bishnoi, P. R. and Svrcek, W. Y. Application of the characteristic method to shock tube data that simulate a gas pipeline rupture. *Can. J. Chem. Eng.* June 1980, **58**(3), 289–294

9 Hartree, D. R. Some Practical Methods of Using Characteristics in the Calculation of Non-steady Compressible Flows. Los Alamos Report LA-HU-1, 1952

10 Lister, M. The numerical solution of hyperbolic partial differential equations by the method of characteristics. *Mathematical Methods for Digital Computers*, by A. Ralston and H. S. Wilf, John Wiley & Sons, 1960, 165–179

11 Roberts, L. On the numerical solution of the equations of spherical waves of finite amplitude II. *J. Math. and Phys.* Jan 1958, **36**(4)

12 Benson, R. S., Garg, R. D. and Woollatt, D. A numerical solution of unsteady flow problems. *Int. J. Mech. Sci.* 1964, **6**(1), 117–144

13 Vardy, A. E. The use of the method of characteristics for the solution of unsteady flows in networks. Presented at the Second International Conference on Pressure Surges, The City University, London, 22–24 September 1976, BHRA, Cranfield, Beds

14 Yow, W. Analysis and Control of Transient Flow in Natural Gas Piping Systems. Ph.D. Thesis, Civil Engineering Dept, University of Michigan, 1971

15 Wylie, E. B. and Streeter, V. L. *Fluid Transients*, FEB Press, Michigan, USA, 1983

16 Forsythe, G. E. and Wasow, W. R. *Finite Difference Methods for Partial Differential Equations*, J. Wiley & Sons, 1960

17 Abarbanel, S., Gottlieb, D. and Turkel, E. Difference schemes with fourth order accuracy for hyperbolic equations. *SIAM J. Appl. Math.* 1975, **29**, 329–351

18 Niessner, H. Comparison of different numerical methods for calculating one-dimensional unsteady flow. Presented at the Von Karman Institute for Fluid Dynamics, Lecture No. 16 from Lecture series 1980–1, *Unsteady One-Dimensional Flows in Complex Networks and Pressurized Vessels*, January 14–18, 1980

19 Lax, P. and Wendroff, B. Systems of conservation laws. *Comm. Pure and Appl. Math.*, 1960, **13**, 217–237

20 Bender, E. Simulation of dynamic gas flows in networks including control loops. *Computers and Chem. Eng.* 1979, **3**, 611 613

21 Martin, C. S., Padmanabhan, M. and Wiggert, D. C. Pressure wave propagation in two-phase bubbly air–water mixtures. Presented at the Second International Conference on Pressure Surges, The City University London, 22–24 September, 1976, BHRA, Cranfield, Beds

22 MacCormack, R. Proceedings of Second International Conference on Numerical Methods in Fluid Dynamics, Lecture Notes in Physics, ed. M. Holt, **8**, Springer-Verlag, New York, 1971

23 Guy, J. J. Computation of unsteady gas flow in pipe networks. Proceedings of Symposium Efficient Methods for Practising Chemical Engineers, Symposium Series No 23, London, Instn. Chem. Engrs. 1967, 139–145

24 Heath, M. J. and Blunt, J. C. Dynamic simulation applied to the design and control of a pipeline network. *Inst. Gas Eng. J.* 1969, **9**(4), 261–279

25 Wilkinson, J. F., Holliday, D. V., Batey, E. H. and Hannah, K. W. Transient Flow in Natural Gas Transmission Systems. Tracor, Inc., published by American Gas Association, New York, 1964

26 Wylie, E. B., Stoner, M. A. and Streeter, V. L. Network system transient calculations by implicit method. *Soc. Pet. Eng. J., Trans. AIME* December 1971, **11**, 356–362

27 Taylor, B. A. The Flow in Pipelines Following Catastrophic Failure. British Gas Internal Report LRS 338, 1978

28 Banerjee, S. and Hancox, W. T. On the development of methods for analysing transient flow boiling. *Int. J. Multiphase Flow*, 1978, **4**, 437–460

29 Chaudhry, M. H. *Applied Hydraulic Transients*, Van Nostrand Reinhold, 1979

30 Padmanabhan, M., Ames, W. F. and Martin, C. S. Numerical analysis of pressure transients in bubbly two-phase mixtures by explicit implicit methods. *J. Eng. Maths.* Jan 1978, **12**(1)

31 Osiadacz, A. Optimal numerical methods for simulating dynamic flow of gas in pipelines. *Int. J. Num. Methods in Fluids* 1983, **3**, 125–135

32 Rachford, H. H. and Dupont, T. A fast highly accurate means of modelling transient flow in gas pipelines by variational methods. *Soc. Pet. Eng. J.* 1974, **14**, 165–178

33 Wathen, A. J. and Baines, M. J. On the Structure of the Moving Finite Element Equations. University of Reading, Numerical Analysis Report No 5/83, 1983

34 Spivack, M. New Algorithms for the Pipebreak Problem. University of Reading report of a feasibility study carried out for British Gas, May–Sept 1984

35 Moretti, G. The λ-scheme. *Computers and Fluids* 1979, **7**(3), 191–205

36 Steger, J. L. and Warming, R. F. Flux vector splitting of the inviscid gasdynamic equations. *J. Computational Phys.* 1981, **40**, 263–293

37 Godunov, S. K. A finite difference method for the numerical computation of discontinuous solutions of the equations of fluid dynamics. *Matematischeskii Sbornik* 1959, **47**, 271–290

38 Mulpuru, S. R. A flux splitting method for the numerical simulation of one-dimensional compressible flow. *Math. and Computers in Simulation* Aug 1983, **25**(4), 309–320

39 Zalesak, S. T. Fully multidimensional flux-corrected transport algorithms for fluids. *J. Computational Phys.* 1979, **31**, 335–362

40 Roe, P. L. and Pike, J. Efficient Construction and Utilisation of Approximate Riemann Solutions. Royal Aircraft Establishment, Bedford, UK, 1984

41 Pandolfi, M. A contribution to the numerical prediction of unsteady flows. AIAA paper 83-0121, presented at the 21st Aerospace Sciences Meeting, Reno, Nev. Jan 10–13 1983

42 Brown, F. T. A quasi-method of characteristics with application to fluid lines with frequency dependent wall shear and heat transfer. *J. Basic Eng., Trans. ASME, Series D* June 1969, **91**(2), 217–227

43 Trikha, A. K. An efficient method for simulating frequency dependent friction in transient liquid flow. *Trans. ASME, J. Fluids Eng., Series I* March 1975, **9**, 97–105

44 Ziele, W. Frequency dependent friction in transient pipe flow. *J. Basic Eng., Trans. ASME, Series D* March 1968, **90**(1), 109–115

45 Henry, L. R. Published discussion at the end of Ref 46. *Trans. ASME, J. Basic Eng.* September 1969, **91**

46 Stoner, M. A. Analysis and control of natural gas piping systems. *Trans. ASME, J. Basic Eng.* September 1969, **91**(3)

47 Oliemans, R. V. A. Two-Phase Flow in Gas Transmission Pipelines. ASME Publication number 76-Pet-25, contributed by the Petroleum Division for presentation at the Joint Petroleum Mechanical Engineering and Pressure Vessels and Piping Conference, Mexico City, Sept 19–24 1976

48 Mathers, W. G., Zuzak, W. W., McDonald, B. H. and Hancox, W. T. On finite difference solutions to the transient flow-boiling equations. Paper presented to the Committee on the Safety of Nuclear Installations Specialists Meeting on Transient Two-phase Flow, Toronto, Ontario, August 1976

49 Kawabe, R. An analytical method for thermal-hydraulic transients in piping networks. *Nucl. Eng. and Design* Dec 1982, **73**(3), 441–446

50 Hancox, W. T. and Nicoll, W. B. A Wall Shear Stress Formula for Adiabatic Two-Phase Flow. Westinghouse Canada Ltd, Report (unpublished), 1972

51 Bakhtar, F. Effects of Wall Friction and Heat Transfer in a Uniform Shock Tube. PhD Thesis, University of Birmingham, 1956

52 Edwards, A. R. and O'Brien, T. P. Studies of phenomena connected with the depressurisation of water reactors. *J. Br. Nucl. Energy Soc.* 1970, **9**, 125–135

53 Groves, T. K., Bishnoi, P. R. and Wallbridge, J. M. E. Decompression wave velocities in natural gases in pipelines. *Can. J. Chem. Eng.* December 1978, **56**, 664–668

54 Williams, A. C. Mass Flow and Velocity in a Shock Tube. Lehigh University Institute of Research, Technical Report No 5, 1956

55 Premoli, A. and Hancox, W. T. An experimental investigation of subcooled blowdown with heat addition. Proc. CSNI Specialists Meeting on Transient Two-Phase Flows, Toronto, Atomic Energy of Canada Ltd, Toronto, 1976

56 Cheeseman, A. P. Application of pressure transient studies to long distance oil and gas transmission lines. Symposium on Pressure Transients held at The City University, Paper No 4, Nov 1970

57 Sens, M., Jouve, Ph. and Pelletier, R. Detection d'une rupture accidentelle de conduite. Paper IGU/C 37-30, presented at the 11th International Gas Conference, Moscow, 1970; (English translation: British Gas Internal Report No LRS, T448)

58 Weimann, A. Gas Distribution Network Dynamic Modelling and Simulation with respect to Network Control and Monitoring. PhD Thesis, Munich Technical University, 1978; (English translation: British Gas Internal Report No LRS T435)

59 Mekebel, S. and Loraud, J. C. Une etude des ecoulement variables dans les conduites de transport de gaz naturel. *Entropie* 1983, **19**(111), 18–25

60 Ying, S. P. and Shah, V. J. Transient pressure in boiler steam lines. Fluid Transients and Acoustics in the Power Industry, presented at the Winter Annual Meeting of the American Society of Mechanical Engineers, San Francisco, Calif. Dec 10–15, 1978

61 Arrison, N. L., Hancox, W. T., Sulatisky, M. T. and Banerjee, S. Blowdown of a recirculating loop with heat addition. Paper 61, Thermodynamics and Fluid Mechanics Group Conference, University of Manchester, 1977

Appendix: Derivation of the basic partial differential equations, with pressure, temperature and velocity as the dependent variables

The basic equations derived from first principles are

$$\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}\right)+\rho\frac{\partial u}{\partial x}=0 \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)+\frac{\partial P}{\partial x}=-\frac{W}{A}-g\sin\theta \tag{2}$$

$$\rho\left(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}\right)-\left(\frac{\partial P}{\partial t}+u\frac{\partial P}{\partial x}\right)=\frac{Wu+\Omega}{A} \tag{3}$$

To obtain ρ in terms of P, z and T

From the equation of state,

$$\rho=\frac{P}{RTz}$$

Therefore

$$\ln\rho=\ln P-\ln R-\ln T-\ln z$$

and, differentiating with respect to time,

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t}=\frac{1}{P}\frac{\partial P}{\partial t}-\frac{1}{R}\frac{\partial R}{\partial t}-\frac{1}{T}\frac{\partial T}{\partial t}-\frac{1}{z}\frac{\partial z}{\partial t}$$

But the compressibility factor $z=z(T, P)$; therefore

$$\mathrm{d}z=\left(\frac{\partial z}{\partial P}\right)_T\mathrm{d}P+\left(\frac{\partial z}{\partial T}\right)_P\mathrm{d}T$$

$$\frac{\mathrm{d}z}{\mathrm{d}t}=\left(\frac{\partial z}{\partial P}\right)_T\frac{\mathrm{d}P}{\mathrm{d}t}+\left(\frac{\partial z}{\partial T}\right)_P\frac{\mathrm{d}T}{\mathrm{d}t}$$

$$\therefore\ \frac{1}{\rho}\frac{\partial \rho}{\partial t}=\left\{\frac{1}{P}-\frac{1}{z}\left(\frac{\partial z}{\partial P}\right)_T\right\}\frac{\mathrm{d}P}{\mathrm{d}t}-\left\{\frac{1}{T}+\frac{1}{z}\left(\frac{\partial z}{\partial T}\right)_P\right\}\frac{\mathrm{d}T}{\mathrm{d}t}$$

Substituting this into Eq (1):

$$\left\{\frac{1}{P}-\frac{1}{z}\left(\frac{\partial z}{\partial P}\right)_T\right\}\frac{\mathrm{d}P}{\mathrm{d}t}-\left\{\frac{1}{T}+\frac{1}{z}\left(\frac{\partial z}{\partial T}\right)_P\right\}\frac{\mathrm{d}T}{\mathrm{d}t}+\frac{\partial u}{\partial x}=0 \tag{A1}$$

To obtain h in terms of P, z and T

From Zemansky[3]:

$$\frac{\mathrm{d}h}{\mathrm{d}t}=C_p\frac{\mathrm{d}T}{\mathrm{d}t}+\left\{\frac{T}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P+1\right\}\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}t}$$

Substituting this into Eq (3):

$$\rho C_p \frac{dT}{dt} + \left\{ \frac{T}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \right\} \frac{dP}{dt} = \frac{\Omega + Wu}{A} \tag{A2}$$

Solving for dP/dt by eliminating dT/dt between (A1) and (A2) yields

$$\rho C_p \left[\frac{1}{P} - \frac{1}{z}\left(\frac{\partial z}{\partial P}\right)_T \right] \frac{dP}{dt} + \left[\frac{1}{T} + \frac{1}{z}\left(\frac{\partial z}{\partial T}\right)_P \right] \frac{T}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p \frac{dP}{dt} = -\rho C_p \frac{\partial u}{\partial x} + \left[\frac{1}{T} + \frac{1}{z}\left(\frac{\partial z}{\partial T}\right)_P \right] \frac{\Omega + Wu}{A} \tag{A3}$$

From the equation of state:

$$\ln \rho = \ln P - \ln R - \ln T - \ln z$$

Differentiating the logarithmic form of the equation of state with respect to temperature T, keeping pressure P constant:

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right) = -\frac{1}{T}\left\{ 1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P \right\}$$

and substituting this into Eq (A3) gives

$$\left\{ \rho C_p \left[\frac{1}{P} - \frac{1}{z}\left(\frac{\partial z}{\partial T}\right)_P \right] - \left[\frac{1}{T} + \frac{1}{z}\left(\frac{\partial z}{\partial T}\right)_P \right]^2 \right\} \frac{dP}{dt} + \rho C_p \frac{\partial u}{\partial x} = \left[\frac{1}{T} + \frac{1}{z}\left(\frac{\partial z}{\partial T}\right)_P \right] \frac{\Omega + Wu}{A}$$

Divide through by C_p:

$$\frac{\rho}{P}\left\{ \left[1 - \frac{P}{z}\left(\frac{\partial z}{\partial P}\right)_T \right] - \frac{PT}{\rho C_p T^2}\left[1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P \right]^2 \right\} \frac{dP}{dt} + \rho \frac{\partial u}{\partial x} = \frac{1}{C_p T}\left[1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P \right] \frac{\Omega + Wu}{A}$$

then solve Eqs (A1) and (A2) for dT/dt:

$$\rho C_p \left[\frac{1}{P} - \frac{1}{z}\left(\frac{\partial z}{\partial P}\right)_T \right] \frac{dT}{dt} + \left[\frac{1}{T} + \frac{1}{z}\left(\frac{\partial z}{\partial T}\right)_P \right] \frac{T}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \frac{dT}{dt} = \left[\frac{1}{P} - \frac{1}{z}\left(\frac{\partial z}{\partial P}\right)_T \right] \frac{\Omega + Wu}{A} + \frac{T}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \frac{\partial u}{\partial x}$$

Dividing through by C_p and substituting for $1/\rho\,(\partial \rho/\partial T)_P$ as before gives:

$$\frac{\rho}{P}\left\{ \left[1 - \frac{P}{z}\left(\frac{\partial z}{\partial P}\right)_T \right] - \frac{PT}{\rho C_p T^2}\left[1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P \right]^2 \right\} \frac{dT}{dt} = \frac{1}{\rho C_p}\left[1 - \frac{P}{z}\left(\frac{\partial z}{\partial P}\right)_T \right] \frac{\Omega + Wu}{A} - \frac{1}{C_p}\left[1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P \right] \frac{\partial u}{\partial x} \tag{A5}$$

Assume entropy s is a function of pressure and density, $s = s(P, \rho)$, then

$$ds = \left(\frac{\partial s}{\partial P}\right)_\rho dP + \left(\frac{\partial s}{\partial \rho}\right)_P d\rho$$

If the entropy is constant then

$$0 = \left(\frac{\partial s}{\partial P}\right)_\rho \left(\frac{\partial P}{\partial \rho}\right)_s + \left(\frac{\partial s}{\partial \rho}\right)_P$$

$$\therefore \left(\frac{\partial P}{\partial \rho}\right)_s = -\left(\frac{\partial s}{\partial T}\right)_P \left(\frac{\partial T}{\partial \rho}\right)_P \Big/ \left(\frac{\partial s}{\partial T}\right)_\rho \left(\frac{\partial T}{\partial P}\right)_\rho$$

Assuming temperature T is a function of pressure and density, $T = T(P, \rho)$, then

$$dT = \left(\frac{\partial T}{\partial P}\right)_\rho dP + \left(\frac{\partial T}{\partial \rho}\right)_P d\rho$$

$$\Rightarrow \left(\frac{\partial T}{\partial \rho}\right)_P \Big/ \left(\frac{\partial T}{\partial P}\right)_\rho = -\frac{1}{\left(\frac{\partial \rho}{\partial P}\right)_T}$$

Therefore

$$\left(\frac{\partial P}{\partial \rho}\right)_s = \left[\left(\frac{\partial s}{\partial T}\right)_\rho \Big/ \left(\frac{\partial s}{\partial T}\right)_\rho \right] \frac{1}{\left(\frac{\partial \rho}{\partial P}\right)_T} \tag{A6}$$

But from Zemansky[3] (p 288):

$$\left(\frac{\partial s}{\partial T}\right)_P = \frac{C_p}{T}$$

and

$$\left(\frac{\partial s}{\partial T}\right)_\rho = \frac{C_v}{T} = \frac{C_p}{T} - \left(\frac{\partial v}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_v$$

Also,

$$\left(\frac{\partial P}{\partial T}\right)_v = -\left(\frac{\partial v}{\partial T}\right)_P \Big/ \left(\frac{\partial v}{\partial P}\right)_T$$

Therefore

$$\left(\frac{\partial s}{\partial T}\right)_\rho = \frac{C_p}{T} - \frac{1}{\rho^2}\left[\left(\frac{\partial \rho}{\partial T}\right)_P^2 \Big/ \left(\frac{\partial \rho}{\partial P}\right)_T \right]$$

and

$$\left(\frac{\partial s}{\partial T}\right)_\rho \left(\frac{\partial \rho}{\partial P}\right)_T = \frac{C_p}{T}\left(\frac{\partial \rho}{\partial P}\right)_T - \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial T}\right)_P^2 \tag{A7}$$

But it has already been proved that

$$\left(\frac{\partial \rho}{\partial T}\right)_P = -\frac{\rho}{T}\left[1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P \right]$$

And, from the equation of state,

$$\left(\frac{\partial \rho}{\partial P}\right)_T = \frac{\rho}{P}\left[1 - \frac{P}{z}\left(\frac{\partial z}{\partial P}\right)_T \right]$$

Substituting these identities into Eq (A6) gives

$$\left(\frac{\partial P}{\partial \rho}\right)_s = \frac{C_p}{T}\left[\frac{C_p}{T}\left\{ \frac{\rho}{P}\left[1 - \frac{P}{z}\left(\frac{\partial z}{\partial P}\right)_T \right] - \frac{1}{C_p T}\left[1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P \right]^2 \right\} \right]^{-1} = \left[\frac{\rho}{P}\left\{ 1 - \frac{P}{z}\left(\frac{\partial z}{\partial P}\right)_T - \frac{P}{\rho C_p T}\left[1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P \right]^2 \right\} \right]^{-1}$$

where $(\partial P/\partial \rho)_s^{1/2}$ can be defined as the Isentropic Wave Speed a_s. Substituting this into Eqs (A4) and (A5):

$$\frac{1}{a_s^2}\frac{dP}{dt} + \rho \frac{\partial u}{\partial x} = \frac{1}{C_p T}\left[1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P \right] \frac{\Omega + Wu}{A}$$

$$\frac{1}{a_s^2}\frac{dT}{dt} + \frac{1}{C_p}\left[1 + \frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P \right] \frac{\partial u}{\partial x} = \frac{1}{C_p P}\left[1 - \frac{P}{z}\left(\frac{\partial z}{\partial P}\right)_T \right] \frac{\Omega + Wu}{A}$$

Finally, rearranging and including Eq (2) gives

$$\frac{\partial P}{\partial t}+u\frac{\partial P}{\partial x}+\rho a_s^2\frac{\partial u}{\partial x}$$
$$=\frac{a_s^2}{C_pT}\left[1+\frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P\right]\frac{\Omega+Wu}{A} \quad (4)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{1}{\rho}\frac{\partial P}{\partial x}=-\frac{W}{\rho A}-g\sin\theta \quad (5)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+\frac{a_s^2}{C_p}\left[1+\frac{T}{z}\left(\frac{\partial z}{\partial T}\right)_P\right]\frac{\partial u}{\partial x}$$
$$=\frac{a_s^2}{C_pP}\left[1-\frac{P}{z}\left(\frac{\partial z}{\partial P}\right)_T\right]\frac{\Omega+Wu}{A} \quad (6)$$

Book review

Convection Heat Transfer

A. Bejan

This text is one of three relatively recently published textbooks dealing with the subject of convection heat transfer. As with the other texts, the book starts by presenting the fundamental laws of conservation of mass, momentum and energy as well as the second law of thermodynamics. Rather than utilizing a generalized control volume and Reynold's transport theorem, the laws are developed from examining a differential control volume in cartesian coordinates. While this leads to easier understanding of the concept by the student, a clear understanding of the stress tensors is lacking and the students may have difficulty in their form in other coordinate systems.

After the first chapter, the text progresses from specialized laminar boundary layer problems to laminar duct flows, natural convection and natural convection in enclosures. The development of laminar boundary layer heat transfer problems in Chapter 2 starts with a general discussion from a physical viewpoint and progresses to integral solutions and then similarity solutions. Practically nothing is presented except for the flat plate solutions, thus the instructor or student will be left to develop the extension to more practical conditions. There are no developments which account for wall suction or blowing on logical extensions to multiphase flows such as film boiling or condensation and so the second chapter is weak in its presentations.

Chapter 3 on laminar duct flow problems introduces first the hydrodynamic entrance length problem prior to going on to fully developed flows. The integral solution between two parallel plates due to Sparrow is discussed but not the more accurate techniques due to L. S. Han or Langhaar. In discussing the heat transfer in ducts, the author starts from the fully developed flow, fully developed temperature profile cases as per Kays and Crawford instead of developing the solutions from the Graetz type problems from where it could be seen that the fully developed solutions come either from the particular solutions or lowest eigenvalues for the case of constant wall temperature of these more general solutions. The thermally developing solutions are also lightly treated and variable thermal properties are totally ignored.

Chapter 4 deals with natural convection and again some classical solutions for a vertical flat plate are introduced, but here discussion of problem formulation for both integral and similarity solutions are more complete. The author makes a false statement concerning lack of understanding of length scales in contemporary research. Such scales have been clearly pointed out by S. Ostrach and B. Gebhart among others. Both high and low Prandtl number solutions for constant wall temperature and constant heat flux are presented. The author also discusses the effects of thermally stratified flows, but erroneously reports the lack of a similarity solution. The existence of such a solution was recently reported by Kulkarni, Jacobs and Hwang, International Journal of Heat and Mass Transfer (1986). Other topics covered include conjugate problems, vertical channel flow and combined natural and forced convection and surprisingly a two-phase problem, gravity driven film condensation, which was lacking in Chapter 2. The latter was only weakly represented by the early pioneering work of Nusselt (1916). The excellent pioneering work of Andrea Acrivos on combined free and forced convection was ignored.

Chapter 5 is a relatively complete treatment of laminar natural convection in closed cells. Some forty pages long, this chapter is an obvious favourite subject of the author who frequently references his own work.

Chapter 6 deals with transition to turbulence and is a good treatment of a topic totally disregarded in most texts. The author is, thus, to be congratulated for its inclusion.

Chapter 7 and 8 on turbulence are reasonably well presented for both forced and free convection. However, the roughly 80 pages seems quite short to treat all aspects of this most important topic of convection heat transfer in a text where two chapters and 73 pages are devoted to flow and heat transfer in porous media!

Despite this book's obvious shortcomings as a textbook in convection heat transfer, it should provide a useful reference source. This is particularly true in the areas of porous media heat transfer which is an area of increasing interest and broadening application.

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